AP PHYSICS B 2009 SUMMER ASSIGNMENT

Please read chapter 3(this handout) and do the Multiple choice and Free Response (open ended) problem at the end of the chapter. Please show all work for the Free Response Problems. All work is to be done neatly. Clearly indicate final answer with units and draw a box around final answer. This will be collected the first day of class.

KINEMATICS IN TWO DIMENSIONS

PREVIEW

Two-dimensional motion includes objects which are moving in two directions at the same time, such as a *projectile*, which has both horizontal and vertical motion. These two motions of a projectile are completely independent of one another, and can be described by *constant velocity* in the horizontal direction, and *free fall* in the vertical direction. Since the two-dimensional motion described in this chapter involves only constant accelerations, we may use the *kinematic equations*.

QUICK REFERENCE

Important Terms

projectile
: any object that is projected by a force and continues to move by its own inertia

range of a projectile
: the horizontal distance between the launch point of a projectile and where it returns to its launch height

trajectory
: the path followed by a projectile

Equations and Symbols

Horizontal direction:

$$v_x = v_{ox} + a_x t$$
$$x = \frac{1}{2}(v_{ox} + v_x)t$$
$$x = v_{ox}t + \frac{1}{2}a_x t^2$$
$$v_x^2 = v_{ox}^2 + 2a_x x$$

$$v_y = v_{oy} + a_y t$$
$$y = \frac{1}{2}(v_{oy} + v_y)t$$
$$y = v_{oy}t + \frac{1}{2}a_y t^2$$
$$v_y^2 = v_{oy}^2 + 2a_y y$$

Vertical direction:

For a projectile near the surface of the earth:

$a_x = 0$, v_x is constant, and $a_y = g = 10$ m/s^2.

DISCUSSION OF SELECTED SECTIONS

Equations of Kinematics in Two Dimensions

Chapter 2 dealt with displacement, velocity, and acceleration in *one dimension*. But if an object moves in the horizontal and vertical direction at the same time, we say that the object is is moving in *two dimensions*. We subscript any quantity which is horizontal with an x (such as v_x and a_x), and we subscript any quantity which is vertical with a y (such as v_y and a_y.)

Example 1 A helicopter moves in such a way that its position at any time is described by the horizontal and vertical equations

$x = 5t + 12t^2$ and $y = 10 + 2t + 6t^2$,

where x and y are in meters and t is in seconds.
(a) What is the initial position of the helicopter at time $t = 0$?
(b) What are the x and y components of the helicopter's acceleration at 3 seconds?
(c) What is the speed of the helicopter at 4 seconds?

Solution:
(a) For the initial position, we simply substitute zero for time:

$x = 5(0) + 12(0)^2$ and $y = 10 + 2(0) + 6(0)^2$
yielding $x = 0$ and $y = 10$ m at $t = 0$.

(b) Notice that both equations are of the familiar form $s = s_0 + v_o t + \frac{1}{2}at^2$. This means that the acceleration in the equation for x must be 24 m/s^2 (that is, ½ $(24)t^2$), and the acceleration in the equation for y must be 12 m/s^2. Thus, $a_x = 24$ m/s^2, and $a_y = 12$ m/s^2.

(c) The velocity in the x – direction v_x would take the form

$v_x = v_{ox} + a_x t = 5 + 24t = 5 + 24(4s) = 101$ m/s.

The velocity in the y – direction would take the form

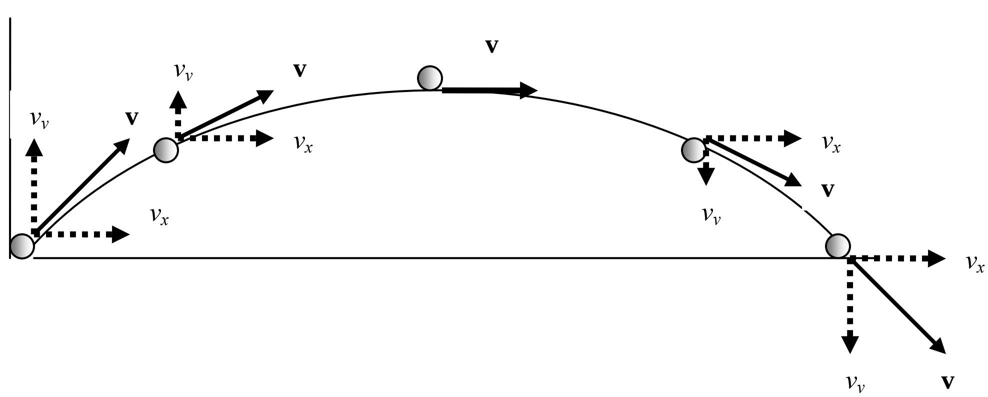
$v_y = v_{oy} + a_y t = 2 + 12t = 2 + 12(4s) = 50$ m/s.
Thus, the speed of the helicopter can be found by Pythagoras' theorem:

$$v = \sqrt{v_x^{\ 2} + v_y^{\ 2}} = \sqrt{(101\,m/s)^2 + (50\,m/s)^2} = 112.7\ m/s$$

3.5 Projectile Motion

Projectile motion results when an object is thrown either horizontally through the air or at an angle relative to the ground. In both cases, the object moves through the air with a constant horizontal velocity, and at the same time is falling freely under the influence of gravity. In other words, the projected object is moving horizontally and vertically at the same time, and the resulting path of the projectile, called the *trajectory*, has a parabolic shape. For this reason, projectile motion is considered to be *two-dimensional* motion.

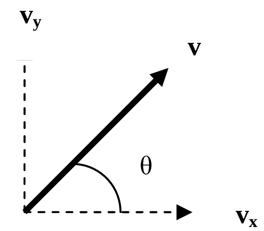
The motion of a projectile can be broken down into constant velocity and zero acceleration in the horizontal direction, and a changing vertical velocity due to the acceleration of gravity. Let's label any quantity in the horizontal direction with the subscript x, and any quantity in the vertical direction with the subscript y. If we fire a cannonball from a cannon on the ground pointing up at an angle θ, the ball will follow a parabolic path and we can draw the vectors associated with the motion at each point along the path:



At each point, we can draw the horizontal velocity vector $\mathbf{v_x}$, the vertical velocity vector $\mathbf{v_y}$, and the vertical acceleration vector **g**, which is simply the acceleration due to gravity.

Notice that the length of the horizontal velocity and the acceleration due to gravity vectors do not change, since they are constant. The vertical velocity decreases as the ball rises and increases as the ball falls. The motion of the ball is symmetric, that is, the velocities and acceleration of the ball on the way up is the same as on the way down, with the vertical velocity being zero at the top of the path and reversing its direction at this point.

At any point along the trajectory, the velocity vector is the vector sum of the horizontal and vertical velocity vectors, that is, $\mathbf{v} = \mathbf{v_x} + \mathbf{v_y}$.



By the Pythagorean theorem,

$$v = \sqrt{{v_x}^2 + {v_y}^2}$$

and

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

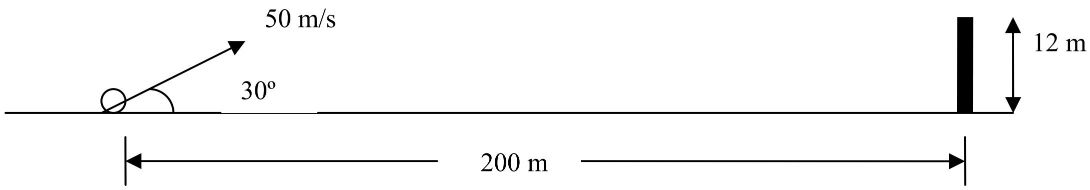
In both the horizontal and vertical cases, the acceleration is constant, being zero in the horizontal direction and 10 m/s^2 downward in the vertical direction, and therefore we can use the kinematic equations to describe the motion of a projectile.

Kinematic Equations for a Projectile

Horizontal motion	**Vertical motion**
$a_x = 0$	$a_y = g = -\ 10$ m/s^2
$v_x = \frac{x}{t}$	$v_y = v_{oy} + g\,t$
$x = v_x t$	$y = v_{oy}t + \frac{1}{2}gt^2$

Notice the minus sign in the equations in the right column. Since the acceleration **g** and the initial vertical velocity $\mathbf{v_{oy}}$ are in opposite directions, we must give one of them a negative sign, and here we've chosen to make *g* negative. Remember, the horizontal velocity of a projectile is constant, but the vertical velocity is changed by gravity.

Example 2 A golf ball resting on the ground is struck by a golf club and given an initial velocity of 50 m/s at an angle of 30º above the horizontal. The ball heads toward a fence 12 meters high at the end of the golf course, which is 200 meters away from the point at which the golf ball was struck. Neglect any air resistance that may be acting on the golf ball.



(a) Calculate the time it takes for the ball to reach the plane of the fence.
(b) Will the ball hit the fence or pass over it? Justify your answer by showing your calculations.
(c) On the axes below, sketch a graph of the vertical velocity v_y of the golf ball vs. time t. Be sure to label all significant points on each axis.

Solution:

(a) The time it takes for the ball to reach the plane of the fence can be found by

$$t = \frac{x}{v_x} = \frac{x}{v\cos\theta} = \frac{200\,m}{(50\,m/s)\cos 30°} = 4.6\,s$$

(b) To determine whether or not the golf ball will strike the fence we need to find the ball's vertical position *y* at the time when the ball is at *x* = 200 m, that is, at 4.6 seconds.

$$y = v_{oy}t + \frac{1}{2}gt^2 = y = v\sin 30t + \frac{1}{2}gt^2 = (50\,m/s)\sin 30(4.6\,s) + \frac{1}{2}(-10\,m/s^2)(4.6\,s)^2$$

$$y = 9.7\,m$$

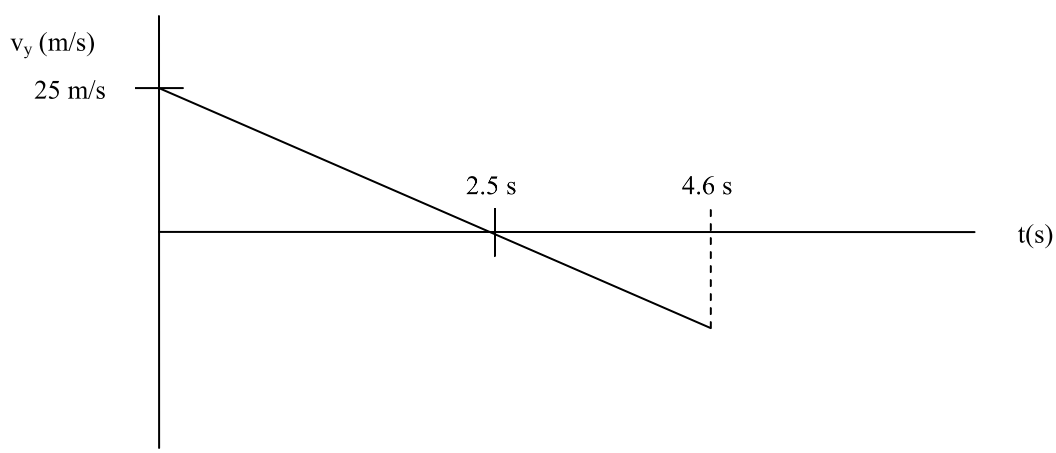
Thus, the ball will strike the fence, since the ball is at a height of less than 12 m when it reaches the plane of the fence.

(c) The *y*-component of the ball's velocity is initially *v*sin 30 = (50 *m/s*) sin 30 = 25 *m/s*. So the vertical speed would begin at 25 *m/s* on the vertical axis, and decrease with a negative slope of 10 m/s^2, crossing the time axis when the vertical velocity is zero, that is, when the ball has reached its maximum height. We can find this time by using the equation

$$v_y = 0 = v_{0y} + gt = v\sin 30 + gt$$

$$t = \frac{v\sin 30}{g} = \frac{(50\,m/s)\sin 30}{10\,m/s^2} = 2.5\,s$$

The ball's vertical velocity is negative (downward) after 2.5 s, until it strikes the fence at 4.6 s.



CHAPTER 3 REVIEW QUESTIONS

For each of the multiple choice questions below, choose the best answer. Unless otherwise noted, use g = 10 m/s² and neglect air resistance.

1. Which of the following is NOT true of a projectile launched from the ground at an angle?
(A) The horizontal velocity is constant
(B) The vertical acceleration is upward during the first half of the flight, and downward during the second half of the flight.
(C) The horizontal acceleration is zero.
(D) The vertical acceleration is 10 m/s^2
(E) The time of flight can be found by horizontal distance divided by horizontal velocity.

2. A projectile is launched horizontally from the edge of a cliff 20 m high with an initial speed of 10 m/s. What is the horizontal distance the projectile travels before striking the level ground below the cliff?
(A) 5 m
(B) 10 m
(C) 20 m
(D) 40 m
(E) 60 m

3. A projectile is launched from level ground with a velocity of 40 m/s at an angle of 30° from the ground. What will be the vertical component of the projectile's velocity just before it strikes the ground? (sin 30° = 0.5, cos 30 = 0.87)
(A) 10 m/s
(B) 20 m/s
(C) 30 m/s
(D) 35 m/s
(E) 40 m/s

Questions 4 – 6
A toy rocket moves in the horizontal direction according to the equation $x = 5t$, and in the vertical direction according to the equation $y = 3t^2$, where x and y are in meters and t is in seconds.

4. The length of the displacement vector of the rocket from the origin (t = 0) at a time of 2 s is most nearly
(A) 22 m
(B) 2 m
(C) – 2 m
(D) 250 m
(E) 16 m

5. The acceleration in the x – direction and the y – direction, respectively, are
(A) zero, 3 m/s^2
(B) zero, 6 m/s^2
(C) 5 m/s^2, 3 m/s^2
(D) 5 m/s^2, 6 m/s^2
(E) 5 m/s^2, 12 m/s^2

6. The horizontal velocity after 10 seconds is most nearly
(A) zero
(B) 5 m/s
(C) 10 m/s
(D) 50 m/s
(E) 300 m/s

Free Response Problem

Directions: Show all work in working the following question. The question is worth 10 points, and the suggested time for answering the question is about 10 minutes. The parts within a question may not have equal weight.

1. (10 points)
Two planetary explorers land on an uncharted planet and decide to test the range of cannon they brought along. When they fire a cannonball with a speed of 100 m/s at an angle of 25˚ from the horizontal ground, they find that the cannonball follows a parabolic path and takes 10 seconds to return to the ground.

(a) Determine the acceleration due to gravity on this uncharted planet.
(b) Determine the maximum height above the level ground the cannonball reaches.
(c) One of the astronauts exclaims that the cannonball "must have landed over a mile away!" Is the astronaut right? Justify your answer (1 mile = 1600 m).
(d) The astronauts then fire another identical cannonball at 100 m/s at an angle of 75˚ to the horizontal ground. Will the cannonball travel a horizontal range x' which is less than, greater than, or equal to the horizontal range for a 25˚ launch angle?

_____ less than _____ greater than _____ equal to

Justify your answer.