PRE AP Physics Summer 2009 Assignment

Please read chapter 1 and do the Multiple choice and Free Response (open ended) problems at the end of the chapter. Please show all work for the Free Response Problems. All work is to be done neatly. Clearly indicate final answer with units and draw a box around final answer. This will be collected the first day of class.

Chapter 1

INTRODUCTION AND MATHEMATICAL CONCEPTS

PREVIEW

This chapter introduces you to the basic mathematical tools for doing physics. You will study *units* and converting between units, the trigonometric relationships of *sine*, *cosine*, and *tangent*, and the manipulation of *scalars* and *vectors*.

QUICK REFERENCE

Important Terms

base SI (Systeme Internationale) units
: the fundamental units of meters for length, kilograms for mass, and seconds for time used in the metric system

cosine
: ratio of the length of the adjacent side to an angle in a right triangle to the length of the hypotenuse

derived unit
: any unit, such as m/s, in the metric system which is not fundamental and is a combination of meters, kilograms, and/or seconds

equilibrant
: a vector which is equal and opposite to the resultant vector, and can cancel the effects of the individual vectors

resultant
: the vector sum of two or more vectors

scalar component
: the magnitude of a component of a vector

scalar quantity
: a quantity, such as mass, which can be completely specified by its magnitude or size.

sine
: ratio of the length of the opposite side to an angle in a right triangle to the length of the hypotenuse

tangent
: ratio of the length of the opposite side to an angle in a right triangle to the length of the adjacent side

vector quantity
a quantity, such as displacement, which is specified by its magnitude (size) and its direction (angle)

vector component
the projection of a vector onto the $x-$ or $y-$ axis

vector addition
adding vectors to each other either graphically (head-to-tail) or using vector components to find the sum (resultant) of the vectors

Equations and Symbols

Rectangle $A = bh$

Triangle $A = \frac{1}{2}bh$

Circle $A = \pi r^2$ and $C = 2\pi r$

Parallelepiped $V = lwh$

Cylinder $V = \pi r^2 l$ and $S = 2\pi rl + 2\pi r^2$

Sphere $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

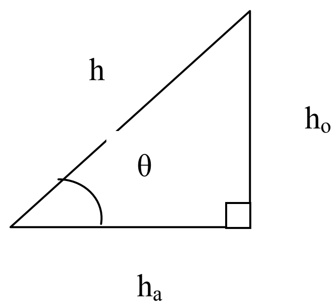
Right Triangle

$h_o^2 + h_a^2 = h^2$

$\sin\theta = \frac{h_o}{h}$

$\cos\theta = \frac{h_a}{h}$

$\tan\theta = \frac{h_o}{h_a}$



where

A = area
b = base
h = height
C = circumference
V = volume
l = length
w = width
S = surface area
r = radius
θ = angle
sin = sine
cos = cosine
tan = tangent

1.3 The Role of Units in Problem Solving

All physical quantities have units so that we can communicate their measurement. IN the metric system, the base units are called SI units. The base SI units for the fundamental quantities of *mass*, *length*, and *time* are the *kilogram*, *meter*, and *second*, respectively. **Only SI units are used on the AP Physics B exam**. Any unit which is a combination of these fundamental units is called a derived unit. An example of a derived unit would be *meters/second* or *kilometers/hour*, which are both units for speed.
Sometimes we will need to convert from one unit to another.

Example 1 Convert 80.0 km/h to m/s.

$$80.0\frac{km}{h}\left(\frac{1000\,m}{1\,km}\right)\left(\frac{1\,h}{3600\,s}\right) = 22.2\frac{m}{s}$$

Note that we multiplied 80.0 km/h by two quotients which were each equal to one, so we didn't change the value of the speed, we only expressed it in different units.

Dimensional Analysis
Often you will need to be able to determine the validity of equations by analyzing the dimensions of the quantities involved.

Example 2 Verify that the equation below is valid by using dimensional analysis.

$$F\Delta t = m\left(v_f - v_0\right)$$

where F is force measured in Newtons, t is time in seconds, m is mass in kg, and v is speed in m/s.
We will show later that a *Newton* = $\frac{kg\,m}{s^2}$. Thus the units for each side of the equation can be written as

$$\left(\frac{kg\,m}{s^2}\right)s = kg\left(\frac{m}{s} - \frac{m}{s}\right)$$

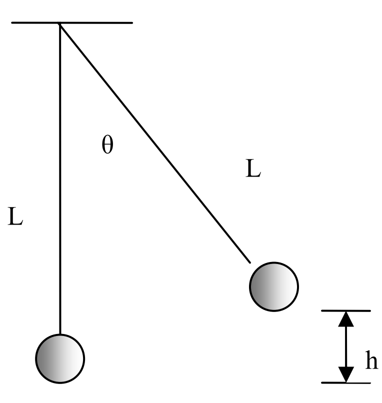
$$\frac{kg\,m}{s} = \frac{kg\,m}{s}$$

The unit $\frac{kg\,m}{s}$ is a unit of momentum or impulse, and will be discussed further in chapter 7.

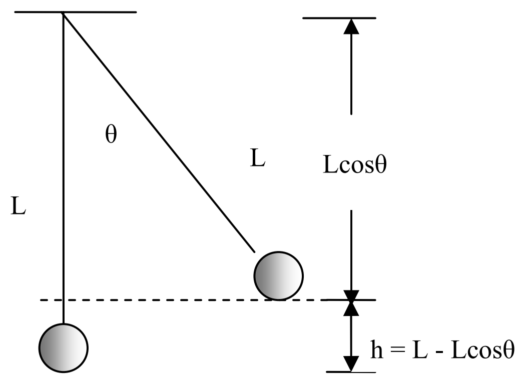
1.4 Trigonometry

Trigonometry is the study of triangles, and often right triangles. The lengths of the sides of a right triangle can be used to define some useful relationships, called the *sine*, *cosine*, and *tangent*, abbreviated *sin*, *cos*, and *tan*, respectively.

Example 3 A ball on the end of a string of length L = 0.50 cm is hung from a hook in the ceiling. The ball is pulled back to an angle θ = 30º from the vertical. What is the height h above the lowest point of the ball?



If we draw a horizontal line from the raised ball to form a right triangle, the portion of the vertical line above the horizontal line forms the adjacent side of the right triangle. This length would then be $L\cos\theta$.



Thus the height *h* would be the difference between *L* and *Lcosθ*:

$$h = L - L\cos\theta = L(1 - \cos\theta)$$

The trigonometric relationships listed in the Equations and Symbols section will be particularly helpful when dealing with vectors.

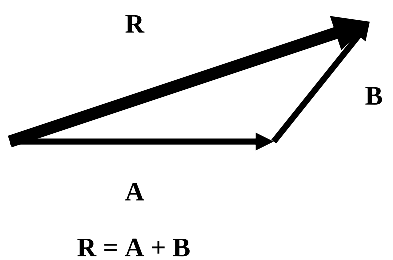
1.5 Scalars and Vectors

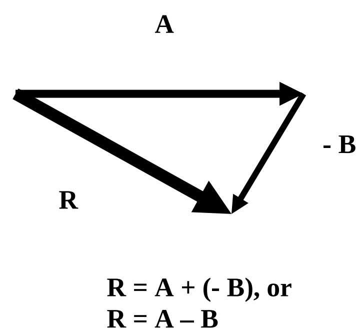
A *scalar* is a quantity which has no direction associated with it, such as mass, volume, time, and temperature. We say that scalars have only magnitude, or size. A mass may have a magnitude of 2 kilograms, a volume may have a magnitude of 5 liters, and so on. But a vector is a quantity which has both magnitude (size) and direction (angle). For example, if someone tells you they are going to apply a 20 pound force on you, you would want to know the direction of the force, that is, whether it will be a push or a pull. So, force is a vector, since direction is important in specifying a force. The same is true of *displacement*, as we will see in the following sections. The table below lists some vectors and scalars you will be using in your physics course.

Vectors	*Scalars*
displacement	*distance*
velocity	*speed*
acceleration	*mass*
force	*time*
weight	*volume*
momentum	*temperature*
	work and energy

1.6 Vector Addition and Subtraction

We can graphically add vectors to each other by placing the tail of one vector onto the tip of the previous vector:





In the diagram on the left above, we have added two vectors head – to – tail by placing the tail of vector **B** on the head (tip) of vector **A**. When adding vectors graphically, we may move a vector anywhere we like, but we must not change its length or direction. The resultant is drawn from the tail of the first vector to the head of the last vector. The resultant is also called the *vector sum* of **A** and **B**, and can replace the two vectors and yield the same result.

Example 4 Displacement is also a vector. Consider a hiker who walks 8 kilometers due east, then 10 km due north, then 12 km due west. What is the hiker's displacement from the origin?

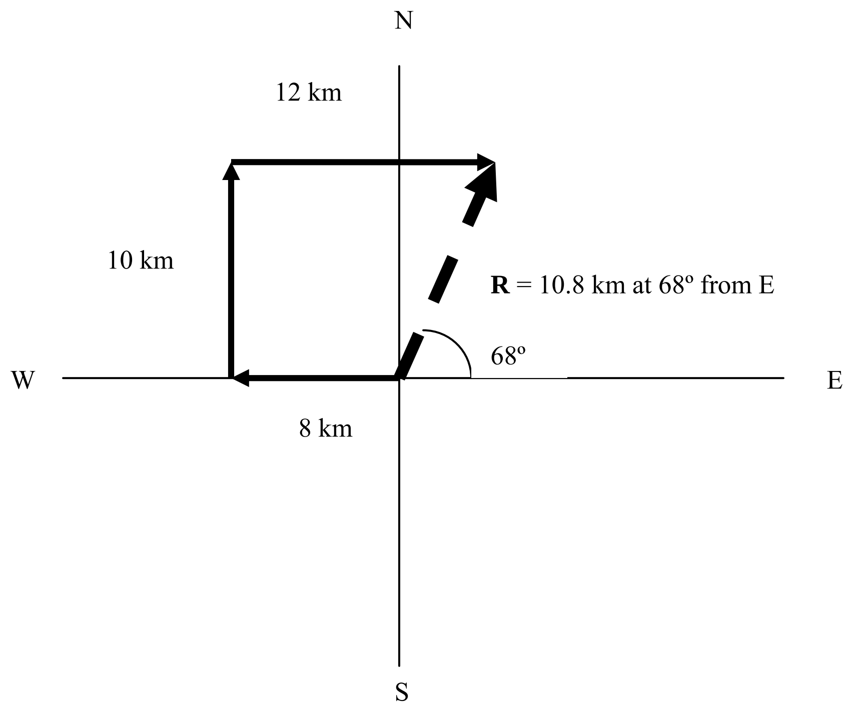
A vector can be represented by an arrow whose length gives an indication of its magnitude (size), with the arrow tip pointing in the direction of the vector. We represent a vector by a letter written in bold type. For this example, we list the displacement vectors like this:

A = 8 km west

B = 10 km north

C = 12 km east

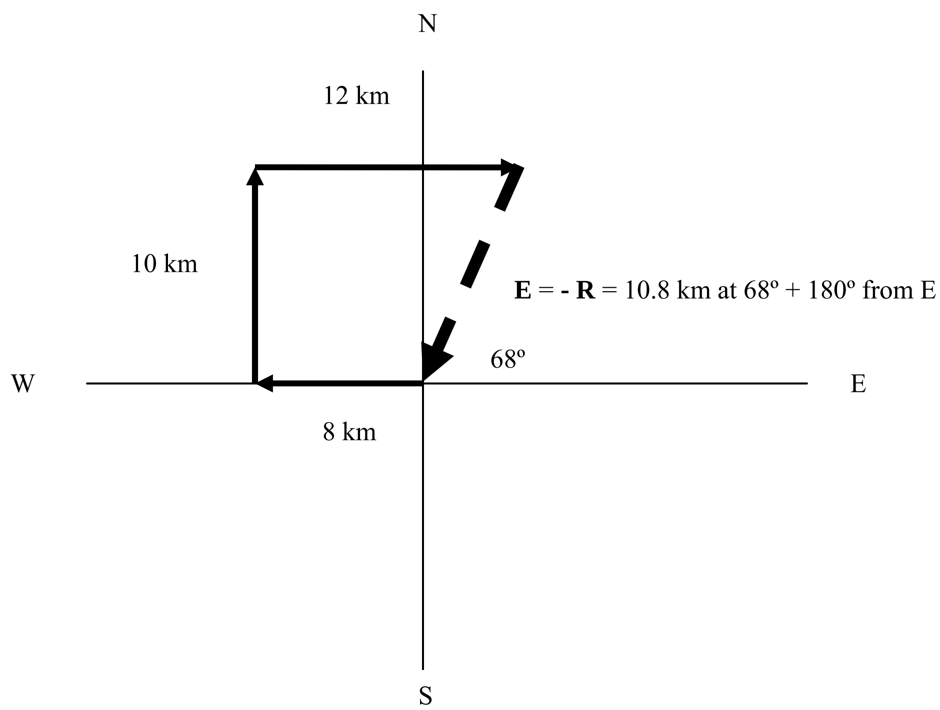
We can graphically add the second displacement vector to the first, and the third displacement vector to the second:



The *resultant vector* is the displacement from the origin to the tip of the last vector. In other words, the *resultant* is the *vector sum* of the individual vectors, and can replace the individual vectors and end up with the same result. Of course, just adding the lengths of the vectors together will not achieve the same result. Adding 8 km, 10 km, and 12 km gives 30 km, which is the total distance traveled, but not the straight-line displacement from the origin. We see in the diagram above that the resultant displacement is 10.8 km from the origin at an angle of 68º from the east axis.

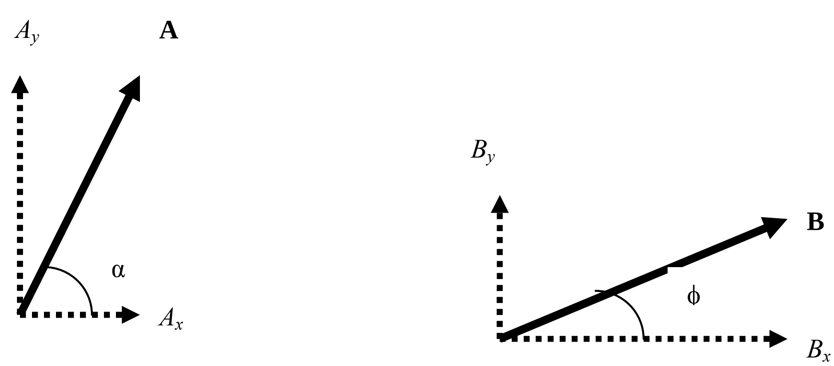
We could have added the displacements in any order and achieved the same resultant. We say that the addition of vectors is *commutative*.

The *equilibrant* is the vector which can cancel or balance the resultant vector. In this case, the equilibrant displacement is the vector which can bring the hiker back to the origin. Thus, the equilibrant is always equal and opposite to the resultant vector.



1.7 The Components of a Vector

We may also work with vectors mathematically by breaking them into their components. A vector component is the projection or shadow of a vector onto the *x*- or *y*-axis. For example, let's say we have two vectors A and B shown below:



We will call the projection of vector **A** onto the x-axis its *x*-component, A_x. Similarly, the projection of **A** onto the *y*-axis is A_y.The vector sum of $\mathbf{A_x}$ and $\mathbf{A_y}$ is **A**, and, since the magnitude of **A** is the hypotenuse of the triangle formed by legs A_x and A_y, the Pythagorean theorem holds true:

$$|A| = \sqrt{A_x^{\ 2} + A_y^{\ 2}}$$

and from the figures above,

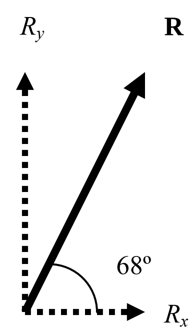
$$A_x = A\sin\alpha$$

$$A_y = A\cos\alpha$$

$$\tan\alpha = \frac{A_y}{A_x}$$

We can write the same relationships for vector **B** by simply replacing *A* with *B* and the angles α with ϕ in each of the equations above.

Example 5 Find the *x*- and *y*-components of the resultant in Example 4.
The resultant is 10.8 km long at an angle of 68º from the east (+*x*) axis:



Since the vector component R_x is adjacent to the 68º angle, we have the relationship

$R_x = R\cos\theta = (10.8\text{ km})\cos 68^\circ = 4\text{ km}$

Similarly, the vector component R_y is opposite to the 68º angle, so we have

$R_y = R\sin\theta = (10.8\text{ km})\sin 68^\circ = 10\text{ km}$

1.8 Addition of Vectors by Means of Components

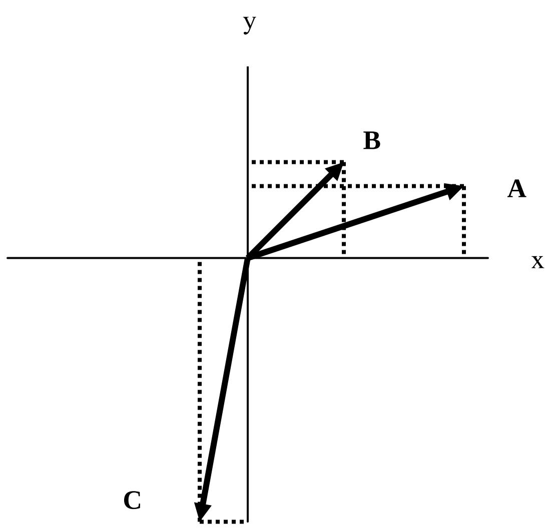
Earlier we added vectors together graphically to find their resultant. Using the head-to-tail method of adding vectors, we can find the resultant of **A** and **B**, which we called **R**. We can also use components to find the resultant of any number of vectors. For example, the *x*-components of the resultant vector **R** is the sum of the *x*-components of **A, B,** and **C**. Similarly, The *y*-components of the resultant vector **R** is the sum of the *y*-components of **A** and **B**.

So, we have that

$R_x = A_x + B_x + C_x$ *and* $R_y = A_y + B_y + C_y$

and by the Pythagorean theorem,

$$|R| = \sqrt{R_x^{\;2} + R_y^{\;2} + R_z^{\;2}}$$



Example 6: Using the diagrams above, let **A** = 4 meters at 30º from the *x*-axis, **B** = 3 meters at 45º from the *x*-axis, and **C** = 5 m at 25º from the *y*-axis. Find the magnitude and direction of the resultant vector **R**. (cos30 = 0.87, sin 30 = 0.50, cos45 = sin45 = 0.70, cos25 = 0.90, sin25 = 0.42)

Solution: First, we need to find A_x, A_y, B_x, B_y, C_x, and C_y :

$A_x = A\cos 30 = (4\text{ m})\cos 30 = (4\text{ m})(0.87) = 3.5\text{ m}.$

$A_y = A\sin 30 = (4\text{ m})\cos 30 = (4\text{ m})(0.50) = 2.0\text{ m}$

$B_x = B\cos 45 = (3\text{ m})\cos 45 = (3\text{ m})(0.70) = 2.1\text{ m}$

$B_y = B\sin 45 = (3\text{ m})\sin 45 = (3\text{ m})(0.70) = 2.1\text{ m}$

$C_x = -\ C\cos 25 = (5\text{ m})\cos 25 = -\ (5\text{ m})(0.90) = -\ 4.5\text{ m}$

$C_y = -\ C\sin 25 = (5\text{ m})\sin 25 = -\ (5\text{ m})(0.43) = -\ 2.2\text{ m}$

Note that C_x and C_y are both negative, since they point to the left and down, respectively. Now we can find the *x*- and *y*-components of the resultant **R**:

$R_x = A_x + B_x + C_x$ = 3.5 m + 2.1 m + (- 4.5 m) = 1.1 m

$R_y = A_y + B_y + C_y$ = 2.0 m + 2.1 m + (- 2.2 m) = 1.9 m

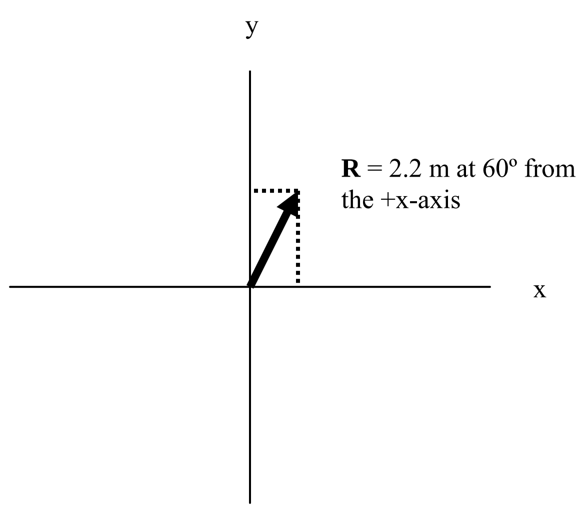
The magnitude of the resultant **C** is

$$|R| = \sqrt{R_x^{\ 2} + R_y^{\ 2}} = \sqrt{(1.1m)^2 + (1.9m)^2} = 2.2m$$

and its angle from the *x*-axis can be found by

$$\tan\theta = \frac{R_y}{R_x}$$

$$\theta = \tan^{-1}\left[\frac{1.9m}{1.1m}\right] = 60°$$



The properties of vectors we've discussed here can be applied to any vector, including velocity, acceleration, force, and momentum.

CHAPTER 1 REVIEW QUESTIONS

For each of the multiple choice questions below, choose the best answer.

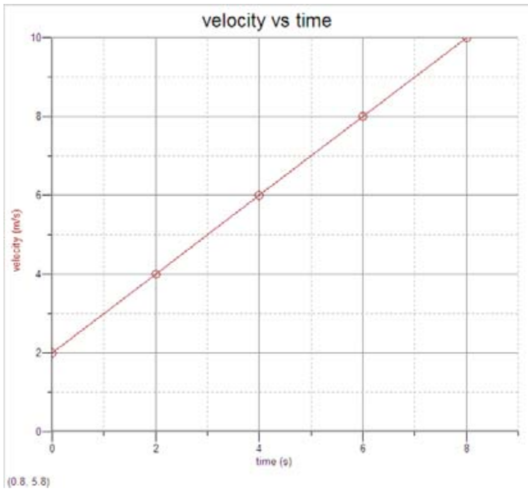
1. The fundamental SI units for mass, length, and time, respectively are

(A) newton, meter, minute
(B) kilogram, meter, second
(C) pound, foot, hour
(D) pound, foot, second
(E) kilogram, centimeter, hour

2. The data plotted on a graph of distance on the *y*-axis vs time on the *x*-axis yields a linear graph. The slope of the graph is

(A) $\frac{\Delta d}{\Delta t}$
(B) $(\Delta d)(\Delta t)$
(C) $\frac{\Delta t}{\Delta d}$
(D) $(\Delta d) + (\Delta t)$
(E) $(\Delta d) - (\Delta t)$

3. Consider the velocity vs time graph shown.



The area under the graph from 2 s to 6 s is most nearly
(A) 8
(B) 16
(C) 24
(D) 32
(E) 36

4. Data from an experiment shows that potential energy is proportional to the square of displacement. The plot of the graph of potential energy (*y*) vs. displacement (*x*) would be

(A) a straight horizontal line
(B) a straight diagonal line sloping upward
(C) a parabola
(D) a hyperbola
(E) a circle

5. The cosine of an acute angle in a right triangle is equal to the

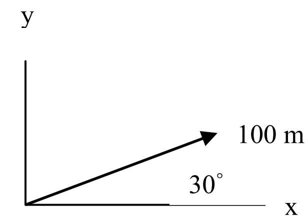
(A) sine of the angle
(B) tangent of the angle
(C) the hypotenuse of the triangle
(D) the side adjacent to the angle
(E) the ratio of the side adjacent to the angle and the hypotenuse of the triangle

6. Which of the following quantities is NOT a vector quantity?

(A) displacement
(B) mass
(C) resultant
(D) equilibrant
(E) 10 km at 30° north of east

7. The resultant of the two displacement vectors 30 m east and 40 m south is
(A) 50 m southeast
(B) 70 m northeast
(C) 10 m southwest
(D) 10 m northeast
(E) 120 m northeast

8.



The *x*-component of the vector shown above is most nearly
(sin 30 = 0.5, cos 30 = 0.87, tan 30 = 0.58)

(A) 100 m
(B) 50 m
(C) 87 m
(D) 58 m
(E) 1000 m

9. Two displacement vectors each having a *y*-component of 100 km are added together vectorially to form a resultant which forms an angle of 60° from the +*x*-axis. What is the magnitude of the resultant? (sin 60° = 0.87, cos 60° = 0.5)

(A) 230 m
(B) 400 m
(C) 120 m
(D) 200 m
(E) 300 m

Free Response Question

Directions: *Show all work in working the following question.*
1. (15 points)

A displacement is a length with a particular direction, and therefore can be represented by a vector. An explorer travels a displacement of 20 km at an angle of 30° from east, then 30 km at 45° from east, then 20 km 120° from east, and finally, 10 km at 240° from east. Label each of these displacement vectors **A**, **B, C,** and **D**, respectively. All angles are measured counterclockwise from the east axis.

(a) Determine the total distance traveled by the explorer in meters.

(b) What is the resultant displacement of the explorer from his original starting point? Be sure to indicate both the magnitude of the resultant displacement and its direction from east.

Resultant ____

(c) Using the sine, cosine, and/or tangent of the appropriate angles, find the *x*- and *y*-components of each of the displacement vectors.

(d) Using addition of vectors by means of components, find the magnitude and direction of the resultant vector.